**Complex number 3**

**1.** Given that , find the values of **(a)** **(b)** .

 (a

**(a)**

**(b)**

**2.** **(a)** Using deMoivre’s Theorem to show that , where a, b and c are integers to be determined.

 **(b)** Express in terms of , where is not a multiple of π.

 Hence, find the roots of the equation in trigonometric form.

 **(a)** , where

 =

 =

 Compare imaginary part, we have

 **(b)**

 =

 =

 Let ,

 , where , where .

 , where .

 , where k = 0, 1,2,3,4.

 Since is not a root of , , where k = 1,2,3,4.

3. **(a)** Find the roots of .

 **(b)** Show that one of the roots in **(a)** is

 **(c)** Show that **(i)** ,

 **(ii)** .

 **(a)** By de Moirvres’ Theorem,

 , k = 0, 1, 2, 3, 4.

 **(b)** In **(a)**, k = 1,

Put , then , ,

 Put , , .

 Since , dividing the left-hand side of the cubic equation by , we get

 ,

 Rejecting the negative root, we have .

 Using Pythagoras theorem,

 Hence

 **(c) (i)** **Method 1**

 , by de Moirvres’ Theorem

 **Method 2**

 (Geometric series)

 , by de Moirvres’ Theorem

 **Method 3**

 Since is a root of , .

 **(c) (ii)**

4. Show that is a root if . The root is located on a circle of radius 2 in an Argand diagram and plot all the roots.

 **Method 1**

 Since complex roots occur in pairs, we consider the polynomial:

 Since irrational roots occur in pairs, we consider the polynomial:

 Also, note that,

 So, we get :

 Roots are:

 (in polar form)



 **Method 2**

 .

 Note that : is a root of which is what we want in the first part of the question.

 Also, other roots can be written in rectangular forms easily.

5. Solve the equation .

 Since the given equation is symmetric, divide by , we get

 Put , eq (1) becomes

 When ,

 When ,

6. ABCD is a square with the letters in the anticlockwise order. The points A and B represents and respectively. Find the complex number represented by C and D.

 Let .

 Rotate anticlockwisely by you get :

 Rotate clockwisely by , you get you get :

7. The equation has imaginary roots. Obtain all the roots of the equation and the value of the real constant k.

 Since the given has an imaginary root, are roots.

 Hence is a factor of

 Compare coefficients term,

 Compare coefficients term, (take )

 Compare constant term,

 Compare coefficients term,

 Hence the equation becomes

 The roots are .

**8. (a)** Let , find .

 **(b)** Hence solve the equations : (i)

 (ii) .

 **(a)**

 **(b)(i)**

 or

 **(ii)**

 or

 or

 or

 or , k = 0, 1.

**9.** If , show that and write in similar form.

**10. (a)** Let , find in Cartesian form.

 **(b)** Find in polar form.

 **(c)** Explain, without plotting, how you can represent in Argand diagram.



 **(a)** where

 **(b)**

 **(c)** It is easier to write the answers in polar form in (b)

 and the Argand diagram is easier to draw.

 If we put

 can be plot by rotating anti-clockwisely the vector by an angle and lengthen the radius by a factor of . Similarly by rotating anti-clockwisely by an angle and lengthen the radius of by a factor of , we can get ,… We can get a spiral of points.

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